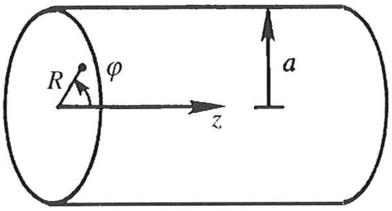
Physics of Fluids (NAPF-12)
May 6, 2014, 18.30 – 21.30 uur

This is a closed-book exam; only simple pocket calculators are allowed.
Put your name and student number on every sheet that you hand in.

Question 1

To investigate arterial blood flow, we analyze a pressure-driven flow of a viscous fluid with density ρ and viscosity μ through a long tube with circular cross-section (radius a). Assume the flow to be axisymmetric and use cylindrical coordinates (R, φ, z), as indicated in the figure. The flow is laminar, fully developed and the only non-zero velocity component is u_z.



10p a) Write down the Navier-Stokes equations in the radial, angular and axial direction and solve them to find u_z [see the Appendix at the end of the exam].

4p b) Calculate the volumetric flow rate through the tube and show that the average velocity can be written as

$$V = -\frac{a^2}{8\mu}\frac{dp}{dz}.$$

[If you were unable to find the answer to question a), make a realistic assumption and use this to solve b)].

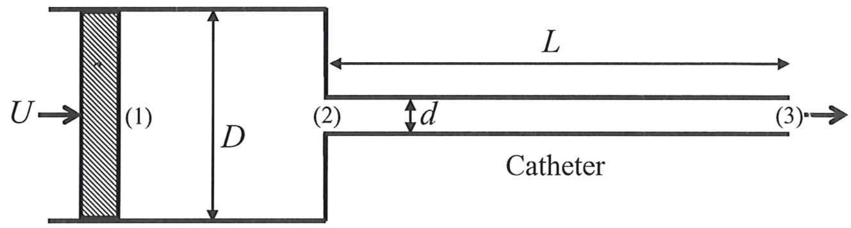
2p c) Endothelial cells that line the inner surface of an artery, can be damaged when the shear stress exceed a critical value. Calculate this shear stress and write it as a function of the average velocity V. [If you were unable to find the answer to question a), make a realistic assumption and use this to solve c)].

Question 2

The figure below shows a schematic representation of a catheter of length L = 200 mm and diameter d = 2 mm attached to a horizontal syringe (NL: injectiespuit) with diameter D = 16 mm. The piston moves with a constant speed U = 1 cm/s to the right. The system is filled with a fluid of density ρ = 1000 kg/m^3 and viscosity μ = 0.04 Ns/m^2. The fluid can be assumed to be incompressible.

First we analyze the syringe without the catheter. In this case, the pressure at the exit of the syringe, point (2), is equal to the atmospheric pressure (p_{atm} = 10^5 N/m^2) and viscosity effects can be neglected.

3p a) The pressure difference Δp between point (1) right next to the piston and point (2) at the syringe exit is a function of D, U, d and ρ. Use Buckingham's Π-theory to derive a set of dimensionless Π-terms that describes this dependence.



2p b) Calculate the velocity U_2 and volumetric flow rate Q of the fluid at the exit (point (2)).

4p c) Calculate the pressure difference Δp.

2p d) What is the pressure right next to the piston at point (1)? Determine the force exerted by the fluid on the piston.

2p e) The solution of this problem can be written in terms of dimensionless Π-terms as $\Pi_1 = \Phi\,(\Pi_2, \Pi_3, ..., \Pi_n)$. Use the Π-terms derived at a) and determine Φ.

Next, we attach the catheter to the syringe (as shown in the figure). In this case the pressure at the catheter exit, point (3), is equal to the atmospheric pressure. The effect of viscosity cannot be neglected in the catheter. Assume that the flow in the catheter is fully developed so that entrance effects at the left side of the catheter can be neglected.

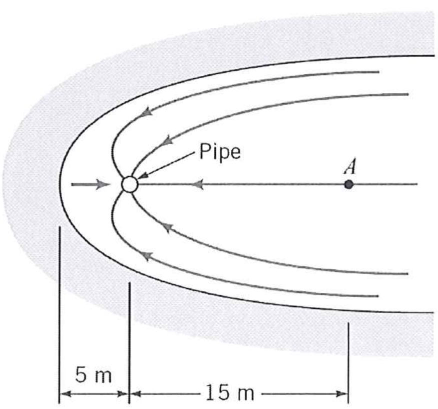
2p f) Is the flow in the catheter laminar or turbulent?

2p g) What is the pressure difference between the entrance (point (2)) and exit (point (3)) of the catheter? [Hint: use your results from Question 1].

2p h) Finally, calculate the pressure at point (1) right next to the piston. What is the force exerted by the piston on the fluid? Is this force larger or smaller compared to the force in absence of the catheter? Explain your answer.

Question 3

One end of a pond (NL: vijver) has a shoreline that resembles a half-body as shown in the figure. The water in the pond has density $\rho = 1000$ kg/m^3. A vertical pipe is located near the end of the pond so that water can be pumped out. The pipe is located



at a distance 5 m from the left side of the pond. We will use a Cartesian frame of reference with the origin located at the position of the pipe. This two-dimensional problem can be analyzed using the following velocity potential $\phi(x,y)$:

$$\phi = -Ux - \frac{m}{2\pi}\ln\sqrt{x^2+y^2}.$$

2p a) This potential is a superposition of two potentials corresponding to elementary flows. Which elementary flows are these?

4p b) Show that the flow described by ϕ is incompressible and irrotational.

4p c) Rewrite $\phi(x,y)$ in terms of polar coordinates (r, θ) and use this to derive the corresponding stream function:

$$\psi = -Ur\sin\theta - \frac{m}{2\pi}\theta.$$

$$[u_r = \frac{\partial\phi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta},\ u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -\frac{\partial\psi}{\partial r}].$$

4p d) Determine the location of the stagnation point (in Cartesian or polar coordinates). Write your answer in terms of U and m.

2p e) The pipe pumps water at such a rate that $m = 0.02\ m^2/s$. Calculate U.

2p f) Calculate the velocity at point A, located at $(x,y) = (15m, 0)$.

2p g) Finally, determine the pressure difference between point A and point $(x,y) = (-5m, 0)$, located at the shore of the pond.

Appendix: Cylindrical coordinates

Equation of continuity: $\frac{\partial\rho}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\rho u_R) + \frac{1}{R}\frac{\partial}{\partial\varphi}(\rho u_\varphi) + \frac{\partial}{\partial z}(\rho u_z) = 0$

Navier-Stokes equations with constant ρ, constant ν, and no body force:

$$\frac{\partial u_R}{\partial t} + (\mathbf{u}\cdot\nabla)u_R - \frac{u_\varphi^2}{R} = -\frac{1}{\rho}\frac{\partial p}{\partial R} + \nu\left(\nabla^2 u_R - \frac{u_R}{R^2} - \frac{2}{R^2}\frac{\partial u_\varphi}{\partial\varphi}\right),$$

$$\frac{\partial u_\varphi}{\partial t} + (\mathbf{u}\cdot\nabla)u_\varphi + \frac{u_R u_\varphi}{R} = -\frac{1}{\rho R}\frac{\partial p}{\partial\varphi} + \nu\left(\nabla^2 u_\varphi + \frac{2}{R^2}\frac{\partial u_R}{\partial\varphi} - \frac{u_\varphi}{R^2}\right),$$

$$\frac{\partial u_z}{\partial t} + (\mathbf{u}\cdot\nabla)u_z = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 u_z,$$

where: $\mathbf{u}\cdot\nabla = u_R\frac{\partial}{\partial R} + \frac{u_\varphi}{R}\frac{\partial}{\partial\varphi} + u_z\frac{\partial}{\partial z}$ and $\nabla^2 = \frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial}{\partial R}) + \frac{1}{R^2}\frac{\partial^2}{\partial\varphi^2} + \frac{\partial^2}{\partial z^2}$.

Strain rate S_{ij} and viscous stress σ_{ij} for an incompressible fluid where $\sigma_{ij} = 2\mu S_{ij}$:

$$S_{RR} = \frac{\partial u_R}{\partial R} = \frac{1}{2\mu}\sigma_{RR},\ S_{\varphi\varphi} = \frac{1}{R}\frac{\partial u_\varphi}{\partial\varphi} + \frac{u_R}{R} = \frac{1}{2\mu}\sigma_{\varphi\varphi},\ S_{zz} = \frac{\partial u_z}{\partial z} = \frac{1}{2\mu}\sigma_{zz}$$

$$S_{R\varphi} = \frac{R}{2}\frac{\partial}{\partial R}\left(\frac{u_\varphi}{R}\right) + \frac{1}{2R}\frac{\partial u_R}{\partial\varphi} = \frac{1}{2\mu}\sigma_{R\varphi},\ S_{\varphi z} = \frac{1}{2R}\frac{\partial u_z}{\partial\varphi} + \frac{1}{2}\frac{\partial u_\varphi}{\partial z} = \frac{1}{2\mu}\sigma_{\varphi z},$$

$$S_{zR} = \frac{1}{2}\left(\frac{\partial u_R}{\partial z} + \frac{\partial u_z}{\partial R}\right) = \frac{1}{2\mu}\sigma_{zR}$$

N.B. For all three questions: when for some reason you are unable to answer a part of a question (a, b, etc.), make a realistic assumption and use this for the rest of the question.

GOOD LUCK!